

# Spacetime symmetries of the Lorentz-violating Maxwell–Chern–Simons model

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The spacetime symmetries of classical electrodynamics supplemented with a Chern–Simons term that contains a constant nondynamical 4-vector are investigated. In addition to translation invariance and the expected three remaining Lorentz symmetries characterized by the little group of the external vector, the model possesses an additional spacetime symmetry if the nondynamical vector is lightlike. The conserved current associated with this invariance is determined, and the symmetry structure arising from this invariance and the usual little group  $\text{ISO}(2)$  is identified as  $\text{SIM}(2)$ .

## I. INTRODUCTION

Lorentz and CPT violation has recently received substantial attention as a potential signature for underlying physics, possibly arising from the Planck scale [1]. The prototype of a Lorentz- and CPT-breaking field theory consists of a three-dimensional Chern–Simons term embedded in Maxwell’s four-dimensional classical electrodynamics [2]. For example, such a term is part of the Standard-Model Extension (SME), the general field-theory framework for Lorentz and CPT tests, which contains the Standard Model of particle physics and general relativity as limiting cases [3]. Numerous experimental and theoretical analyses of Lorentz and CPT breakdown have been performed within the SME.

Astrophysical spectropolarimetry constrains the Lorentz and CPT violation described by the Maxwell–Chern–Simons model to an extraordinary degree [2], so that it can be set to zero for all practical purposes. Nevertheless, this model continues to enjoy a unique popularity for theoretical investigations, for it is simple, long-established, and mathematically interesting. Examples of recent studies in the context of the Maxwell–Chern–Simons model include ones involving radiative corrections [4], nontrivial spacetime topology [5], causality [6], energy positivity [7], supersymmetry [8], vacuum Čerenkov radiation [9], and the cosmic microwave background [10]. More recently, the idea behind the construction of the Chern–Simons term in electrodynamics has also been applied to obtain a similar Lorentz-violating extension of general relativity [11].

The present note continues along these lines and employs the Maxwell–Chern–Simons model as a simple theoretical laboratory to study the number of violated spacetime symmetries in the presence of an external 4-vector. Our results confirm that typically the Lorentz group is broken down to the little group associated with the external 4-vector. However, in certain circumstances involving the special case of a lightlike external vector, a further continuous symmetry relative to the timelike and space-

like cases may exist. In such situations, an additional conserved current exists, which often simplifies practical applications and may yield insight into the structure of the model.

This paper is organized as follows. Section II reviews the basics of the Maxwell–Chern–Simons model. In Sec. III, we perform a general killing-vector analysis to identify the remaining spacetime symmetries of this model, and establish that a lightlike Lorentz violation maintains one more symmetry relative to the timelike and spacelike cases. The associated conserved current is constructed in Sec. IV. Section V determines the resulting larger symmetry group. A brief summary is contained Sec. VI.

## II. PHOTONS WITH A CHERN–SIMONS TERM

This section reviews various results concerning the Lorentz- and CPT-violating Chern–Simons extension of electrodynamics. In natural units  $c = \hbar = 1$ , the model Lagrangian in the presence of external sources  $j^\mu$  is given by

$$\mathcal{L}_{\text{MCS}} = -\frac{1}{4}F^2 + k_\mu A_\nu \tilde{F}^{\mu\nu} - A \cdot j. \quad (1)$$

Here,  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  and its dual  $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$  are defined as usual. The nondynamical fixed  $k^\mu$  determines a preferred direction in spacetime violating Lorentz as well as CPT symmetry. Although this Lagrangian is gauge dependent, the associated action integral is invariant if the source  $j^\mu$  is conserved.

The Lagrangian (1) leads to the following equations of motion for the potentials  $A^\mu = (A^0, \vec{A})$ :

$$(\square\eta^{\mu\nu} - \partial^\mu\partial^\nu - 2\varepsilon^{\mu\nu\rho\sigma}k_\rho\partial_\sigma)A_\nu = j^\mu. \quad (2)$$

As in conventional electrodynamics, current conservation  $\partial_\mu j^\mu = 0$  follows as a consistency requirement. The resulting modified Maxwell equations

$$\partial_\mu F^{\mu\nu} + 2k_\mu \tilde{F}^{\mu\nu} = j^\nu \quad (3)$$

are gauge invariant, as expected. For completeness, we also display the modified Coulomb and Ampère laws contained in Eq. (3):

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} - 2\vec{k} \cdot \vec{B} &= \rho, \\ -\dot{\vec{E}} + \vec{\nabla} \times \vec{B} - 2k_0 \vec{B} + 2\vec{k} \times \vec{E} &= \vec{j}. \end{aligned} \quad (4)$$

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The homogeneous Maxwell equations remain unaltered because the field–potential relationship is conventional.

The Lagrangian (1) is invariant under spacetime translations, and therefore a conserved energy–momentum tensor  $\Theta^{\mu\nu}$  can be constructed. Starting from the canonical expression and adding judiciously chosen superpotential terms  $\partial_\alpha X^{[\alpha\mu]\nu}$ , the relatively compact form

$$\Theta^{\mu\nu} = \frac{1}{4} \eta^{\mu\nu} F^2 - F^{\mu\alpha} F^\nu{}_\alpha - k^\nu \tilde{F}^{\mu\alpha} A_\alpha \quad (5)$$

can be derived. Here,  $\eta^{\mu\nu}$  denotes the usual metric with signature  $-2$ . In the absence of sources  $j^\mu = 0$ , this tensor is conserved in its  $\mu$  index:  $\partial_\mu \Theta^{\mu\nu} = 0$ . Note that the usual Belinfante symmetrization procedure is inapplicable because Lorentz symmetry is violated. Note also that  $\Theta^{\mu\nu}$  is gauge dependent. But the additional term  $-k^\nu \tilde{F}^{\mu\alpha} \partial_\alpha \Lambda = -\partial_\alpha k^\nu \tilde{F}^{\mu\alpha} \Lambda$  generated by a gauge transformation  $A_\alpha \rightarrow A_\alpha + \partial_\alpha \Lambda$  is a superpotential, which leaves unaffected the conserved 4-momentum  $P^\nu \equiv \int d^3x \Theta^{0\nu}$ .

The ansatz  $A^\mu(x) = \epsilon^\mu(\lambda) \exp(-i\lambda \cdot x)$ , where  $\lambda^\mu \equiv (\omega, \vec{\lambda})$ , together with the equations of motion (2) yields the plane-wave dispersion relation:

$$\lambda^4 + 4\lambda^2 k^2 - 4(\lambda \cdot k)^2 = 0. \quad (6)$$

This equation determines the wave frequency  $\omega$  for a given wave 3-vector  $\vec{\lambda}$ . For a timelike  $k^\mu$ , the magnitude of the group velocity determined by the dispersion relation (6) can exceed the light speed  $c$ . Indeed, previous analyses have established theoretical difficulties associated with instabilities and causality violations for  $k^2 > 0$ . These issues are absent for  $k^2 \leq 0$ . In what follows, we focus primarily on the case of a lightlike  $k^\mu$  in the absence of sources  $j^\mu = 0$ .

### III. REMAINING SPACETIME SYMMETRIES

Besides the usual ten Poincaré invariances associated with four translations, three rotations, and three boosts, the conventional free Maxwell field possesses five additional spacetime symmetries arising from one dilatation

and four special conformal transformations. The inclusion of the Chern–Simons term  $\mathcal{L}_{\text{CS}} \equiv k^\alpha A^\beta \tilde{F}_{\alpha\beta}$  maintains translation invariance, since  $k^\mu$  is assumed to be constant. However, one expects that dilatation and conformal symmetry are lost because  $k^\mu$  has mass dimensions setting a definite scale. One further expects that the Lorentz group is broken down to the appropriate little group associated with  $k^\mu$ . For timelike, spacelike, and lightlike  $k^\mu$ , the little groups are  $\text{SO}(3)$ ,  $\text{SO}(2,1)$ , and  $\text{ISO}(2)$ , respectively. Each of these groups is three dimensional, so that at least three of the original six Lorentz symmetries are maintained. This section shows that for a lightlike  $k^\mu$  one additional spacetime symmetry, which is a combination of a dilatation and a boost, exists.

We begin by recalling that a conformal Killing-vector field  $f^\mu(x)$  associated with the four-dimensional Minkowski metric  $\eta^{\mu\nu}$  satisfies

$$\partial^\mu f^\nu + \partial^\nu f^\mu = \frac{1}{2} \eta^{\mu\nu} \partial_\alpha f^\alpha, \quad (7)$$

and possesses the general solution

$$f^\mu = a^\mu + \omega^{\mu\nu} x_\nu + \rho x^\mu + 2(x \cdot b) x^\mu - x^2 b^\mu. \quad (8)$$

Here,  $a^\mu$ ,  $\omega^{\mu\nu} = -\omega^{\nu\mu}$ ,  $c$ , and  $b^\mu$  are free coefficients parametrizing translations, Lorentz transformations, dilatations, and conformal transformations, respectively. With this Killing-vector field we may construct the current

$$J^\mu \equiv \Theta^{\mu\nu} f_\nu. \quad (9)$$

Employing the defining Eq. (7) and energy–momentum conservation  $\partial_\mu \Theta^{\mu\nu}$ , one finds that  $\partial_\mu J^\mu = \frac{1}{4} \Theta^{\mu\nu} \partial_\alpha f^\alpha + \Theta^{\mu\nu} \partial_{[\mu} f_{\nu]}$ . In the conventional Maxwell case, this divergence vanishes because  $\Theta^{\mu\nu}$  is traceless and symmetric confirming the existence of 15 conserved quantities.

In the present Lorentz-violating case,  $\Theta^{\mu\nu}$  possesses a piece that fails to be traceless and symmetric: the term containing  $k^\nu$  in Eq. (5). The Chern–Simons extension therefore leads to

$$\partial_\mu J^\mu = [\omega_{\mu\nu} k^\nu - \rho k_\mu + 2(k \cdot b) x_\mu - 2(k \cdot x) b_\mu - 2(b \cdot x) k_\mu] \tilde{F}^{\mu\alpha} A_\alpha. \quad (10)$$

This is conserved for arbitrary solutions  $A_\alpha$ ,  $\tilde{F}^{\mu\alpha}$  and for arbitrary gauges, if the expression in the brackets vanishes identically. This expression can be viewed as a linear function in the free variable  $x$ , so that two simultaneous conditions emerge:

$$(k \cdot b) \eta^\mu{}_\nu - b^\mu k_\nu - k^\mu b_\nu = 0 \quad (11)$$

from the coefficient in front of  $x$  and

$$\omega_{\mu\nu} k^\nu - c k_\mu = 0 \quad (12)$$

from the  $x$ -independent term. These conditions determine the remaining symmetries.

Condition (11) implies that  $b^\mu = 0$ , which can be established as follows. The trace of Eq. (11) gives  $k \cdot b = 0$ . Us-

ing this fact and contracting Eq. (11) with  $b^\nu$  shows that  $b^2 = 0$ . Similarly, contraction with  $k^\nu$  yields  $b^\mu k^2 = 0$ . Suppose  $b^\mu \neq 0$ , so that  $k^2 = 0$ . Then, the requirements  $k \cdot b = k^2 = b^2 = 0$  imply that  $b^\mu$  and  $k^\mu$  are (anti)parallel. But then, the condition (11) could be cast into the form  $k^\mu k_\nu = 0$ , which is inconsistent with the assumption of a nontrivial Chern–Simons term. It follows that  $b^\mu$  must indeed vanish, so that conformal symmetry is broken in our Maxwell–Chern–Simons model, as expected.

Condition (12) can be satisfied for  $\rho = 0$  and  $\omega^{\mu\nu} k_\nu = 0$ . This means that  $\omega^{\mu\nu}$  must lie in the parameter space of the little group associated with  $k^\mu$ . For example, if  $k^\mu$  is timelike, we may select an inertial coordinate system in which  $k^\mu = (k^0, \vec{0})$ . In this frame, we need  $\omega^{0j} = -\omega^{j0} = 0$  to ensure  $\omega^{\mu\nu} k_\nu = 0$ , i.e., boosts are no longer a symmetry. Here, spatial components are denoted by lower-case Latin indices  $j, k, l$ , etc. However, arbitrary spatial rotations parametrized by  $\omega^{jk} = \epsilon^{jkl} \theta^l$ , which correspond to the little group  $\text{SO}(3)$ , remain compatible with  $\omega^{\mu\nu} k_\nu = 0$ . A similar reasoning applies to the cases of a spacelike and lightlike  $k^\mu$  with their respective little groups  $\text{SO}(2,1)$  and  $\text{ISO}(2)$ . This  $\rho = 0$  solution to the condition (12) is expected and unsurprising.

We may also ask whether condition (12) can be satisfied for the case  $c \neq 0$ . Contraction of Eq. (12) with  $k^\mu$  shows that  $k^2 = 0$  is a necessary requirement in this case. Separating the temporal and spatial components of condition (12) yields

$$\vec{\beta} \cdot \hat{k} = c \quad (13)$$

and

$$\vec{\beta} + \vec{\theta} \times \hat{k} = c \hat{k} \quad (14)$$

Here, we have defined  $k^\mu = k(1, \hat{k})$ , where  $\hat{k}$  is a unit 3-vector. In addition, we have denoted the rapidity  $\vec{\beta}$  of a boost by  $\omega_{0j} = \beta^j$  and the angle  $\vec{\theta}$  of a rotation by  $\omega_{jk} = \epsilon^{jkl} \theta^l$ . Equation (14) may be further decomposed into its components parallel and perpendicular to  $\hat{k}$ :

$$\vec{\beta}_\parallel = c \hat{k}, \quad (15a)$$

$$\vec{\beta}_\perp = \hat{k} \times \vec{\theta}. \quad (15b)$$

Here, we have set  $\vec{\beta}_\parallel \equiv \vec{\beta} \cdot \hat{k}$  and  $\vec{\beta}_\perp \equiv \vec{\beta} - \vec{\beta}_\parallel \hat{k}$ . We remark in passing that Eq. (15a) is equivalent to Eq. (13), which means Eq. (13) is also contained in Eq. (14). Equation (15b), which does not contain  $\rho$ , is associated with the expected three remaining Lorentz symmetries described the little group  $\text{ISO}(2)$  of our lightlike  $k^\mu$ . One of these corresponds to rotations  $\vec{\theta} = \theta \hat{k}$  about  $\hat{k}$ . The other two correspond to the two possible independent boosts in the plane orthogonal to  $\hat{k}$ , which must be simultaneously performed together with the appropriate rotation about an axis perpendicular to both the boost direction and  $\hat{k}$ . In addition to this standard result for the little group of a lightlike 4-vector, there is one more symmetry in the present case: according to Eq. (13), or equivalently Eq.

(15a), a boost along the direction of  $\hat{k}$  together with an appropriate dilatation also satisfies condition (12). In the next section, we discuss this additional invariance further.

#### IV. ADDITIONAL SPACETIME SYMMETRY FOR LIGHTLIKE $k^\mu$

We begin by recalling that a dilatation, which is also called a scale transformation, takes  $x^\mu \rightarrow x_D^\mu = e^{+\rho} x^\mu$  and  $A^\mu \rightarrow A_D^\mu = e^{-\rho} A^\mu$ , where the size of the dilatation is determined by the parameter  $\rho$ . It is apparent that the unconventional Chern–Simons-type term in Lagrangian (1) not only violates Lorentz symmetry, but it also breaks scale invariance because  $k^\mu$  has mass dimensions. To see this explicitly, we decompose the Lagrangian (1) according to  $\mathcal{L} = \mathcal{L}_M + \mathcal{L}_{CS}$ . Here,

$$\mathcal{L}_M = -\frac{1}{4} F^2 \quad (16)$$

denotes the conventional Maxwell piece and

$$\mathcal{L}_{CS} = k^\alpha A^\beta \tilde{F}_{\alpha\beta} \quad (17)$$

the Chern–Simons piece, as before. We remind the reader that we consider the free case  $j^\mu = 0$  only. A dilatation takes

$$\mathcal{L} \rightarrow \mathcal{L}_M + e^{-\rho} \mathcal{L}_{CS} \neq \mathcal{L}. \quad (18)$$

Note that  $x$  becomes a dummy integration variable in the action, so the  $x$  dependence of the fields in the above transformation can be suppressed. We see that the conventional piece and the Chern–Simons extension transform differently. Moreover, the difference between the original and the transformed Lagrangians fails to be a total derivative, which establishes the non-invariance of  $\mathcal{L}$  under dilatations.

We next consider Lorentz transformations, which can be implemented via  $\Lambda^\mu_\nu(\vec{\theta}, \vec{\beta})$ . As before,  $\vec{\theta}$  and  $\vec{\beta}$  characterize rotations and boosts, respectively. Under such transformations, the Lagrangian (1) changes according to  $\mathcal{L} \rightarrow \mathcal{L}_M + \Lambda^\mu_\gamma(-\vec{\theta}, -\vec{\beta}) k^\gamma A^\nu \tilde{F}_{\mu\nu}$  in the absence of sources. We have again suppressed the dependence on the dummy integration variables  $x$  for brevity. Motivated by the discussion in the previous section, we consider a boost along  $\hat{k}$  with rapidity  $\beta$ . Such a transformation changes the magnitude of  $k^\mu$  by a factor of  $e^\beta$ . We then have  $\Lambda^\mu_\gamma(\vec{0}, -\beta \hat{k}) k^\gamma A^\nu \tilde{F}_{\mu\nu} = \exp(\beta) k^\mu A^\nu \tilde{F}_{\mu\nu} \neq \mathcal{L}_{CS}$ , so that

$$\mathcal{L} \rightarrow \mathcal{L}_M + e^\beta \mathcal{L}_{CS} \neq \mathcal{L}, \quad (19)$$

which establishes that symmetry under boosts along  $\hat{k}$  is violated, as expected.

Although each individual transformation (18) and (19) is no longer associated with a symmetry, the specific form

of these transformations and the arguments in the previous section show that a dilatation *combined* with a suitable boost along the spatial direction of a lightlike  $k^\mu$  remains a symmetry of the free part of Lagrangian (1). We can see this explicitly by examining the currents

$$D^\mu \equiv \theta^{\mu\nu} x_\nu \quad (20)$$

and

$$J_{\alpha\beta}^\mu \equiv \theta_\alpha^\mu x_\beta - \theta_\beta^\mu x_\alpha. \quad (21)$$

These quantities are defined to give the usual dilatation and Lorentz currents in the  $k^\mu \rightarrow 0$  limit. To extract from Eq. (21) the current associated with a boost along  $\hat{k}$ , we split  $k^\mu$  into its purely timelike and its purely spacelike part  $k^\mu = k(k_T^\mu + k_S^\mu)$ , where  $k_T^\mu = (1, \vec{0})$  and  $k_S^\mu = (0, \hat{k})$ . The projection onto the desired components is then given by  $J_{\alpha\beta}^\mu k_S^\alpha k_T^\beta$ . The divergences of these currents obey

$$\partial_\mu D^\mu = -\mathcal{L}_{CS} \quad (22)$$

and

$$\partial_\mu J_{\alpha\beta}^\mu k_S^\alpha k_T^\beta = +\mathcal{L}_{CS}. \quad (23)$$

It is again apparent that  $D^\mu$  and  $J_{\alpha\beta}^\mu k_S^\alpha k_T^\beta$  fail to be conserved individually, but their sum  $Q^\mu \equiv D^\mu + J_{\alpha\beta}^\mu k_S^\alpha k_T^\beta$  determines, in fact, a conserved current. An explicit expression for  $Q^\mu$  can be obtained via their definitions (20) and (21) as well as Eq. (5):

$$Q^\mu = \left[ \frac{1}{4} \eta_\nu^\mu F^2 + F^{\mu\alpha} F_{\alpha\nu} \right] [x_\nu + (k_T \cdot x) k_S^\nu - (k_S \cdot x) k_T^\nu]. \quad (24)$$

This expression puts into evidence the manifest gauge invariance of  $Q^\mu$ .

## V. ASSOCIATED SPACETIME-SYMMETRY GROUP

In the previous sections, we have found an additional symmetry for the Maxwell–Chern–Simons model with lightlike Lorentz violation. It is now natural to ask what the full spacetime-symmetry group of this model is. This question is the subject of the present section.

In the case of a lightlike  $k^\mu$ , the little group (i.e., the unbroken subgroup of the Lorentz group) is isomorphic to the three-dimensional Euclidean group ISO(2), which consists of rotations and translations in two dimensions. Since it is always possible to find an inertial coordinate system in which the vector  $k^\mu$  points along the  $z$ -axis, we may assume this choice without loss of generality. The group ISO(2) is then generated by the Lie algebra spanned by the following generators:  $A = J_2 + K_1$ ,  $B = -J_1 + K_2$ , and  $J_3$ . Here,  $J_1, J_2, J_3$  denote the generators of rotations about the  $x, y$ , and  $z$  axes respectively, and  $K_1, K_2, K_3$  generate boosts along the  $x, y$ , and  $z$  axes.

The additional symmetry found in the previous sections corresponds to a combination of a dilatation with a boost in the spatial direction of  $k^\mu$ . This transformation can be generated by an element of the form  $C = K_3 + D$ , where  $D$  is the usual uniform dilatation in the independent and dependent variables. Because of its uniformity, the dilatation  $D$  commutes with all of the rotation and boost operators listed above. Besides translations, the spacetime-symmetry group for the Maxwell–Chern–Simons model with lightlike symmetry breaking is thus generated by

$$A = J_2 + K_1, \quad B = -J_1 + K_2, \quad J_3, \quad C = K_3 + D, \quad (25)$$

where the first three operators are the generators of ISO(2).

For further study of the structure of this symmetry, we determine the commutation relations between the generators (25). To this end, we recall the commutation relations between elements of the usual Lorentz algebra:

$$[J_j, J_k] = i\varepsilon^{jkl} J_l, \quad (26a)$$

$$[J_j, K_k] = i\varepsilon^{jkl} K_l, \quad (26b)$$

$$[K_j, K_k] = -i\varepsilon^{jkl} J_l. \quad (26c)$$

These equations and the fact that  $D$  commutes with all  $J_j$  and  $K_j$  determine the commutation relations of the vector fields (25), which we have summarized in Table I.

TABLE I: Commutation table for the Lie algebra spanned by the vector fields (25).

	A	B	J <sub>3</sub>	C
A	0	0	$-iB$	$iA$
B	0	0	$iA$	$iB$
J <sub>3</sub>	$iB$	$-iA$	0	0
C	$-iA$	$-iB$	0	0

Inspection of the Commutation Table I reveals that the algebra closes. Moreover, these commutation relations can be identified with those of the abstract Lie algebra  $\text{sim}(2)$  of the four-dimensional similitude group of  $\mathbb{R}^2$ , which is explicitly given by  $\text{SIM}(2) = \mathbb{R}^+ \times \mathbb{R}^2 \times \text{SO}(2)$  [12]. In the conventional notation [13], the algebra  $\text{sim}(2)$  is generated by the vector fields  $T_a, T_{b_1}, T_{b_2}, T_\theta$ , and its commutation relations are given by

$$[T_a, T_{b_k}] = -iT_{b_k}, \quad [T_\theta, T_{b_k}] = -i\varepsilon^{3kl} T_{b_l}. \quad (27)$$

Comparison with the Commutation Table I establishes that we may identify  $T_a$  with  $C$ ,  $T_{b_1}$  with  $B$ ,  $T_{b_2}$  with  $A$ , and  $T_\theta$  with  $J_3$ .

We finally remark that the similitude group  $\text{SIM}(2)$  has recently been employed as the starting point for a particular approach to Lorentz violation [14]. From

the perspective of this approach, the present Maxwell–Chern–Simons model represents a specific realization of a SIM(2)-invariant theory. As opposed to the fermion example considered in the SIM(2) approach, our SIM(2)-invariant model does not involve nonlocal operators. We note that the SIM(2) approach is compatible with a subset of the usual supersymmetries [15].

## VI. SUMMARY

This work has investigated the spacetime symmetries in the free Lorentz- and CPT-violating Maxwell–Chern–Simons model. The number of these symmetries depends on the spacetime character of the background vector producing the symmetry breaking: for a lightlike vector one more invariance relative to the timelike and spacelike cases exists. This additional symmetry results from a specific combination of a boost and a dilatation. We

have determined the associated conserved current, which is given in Eq. (24). In Sec. V, we have demonstrated that the usual little group ISO(2) associated with a lightlike vector is enlarged to the similitude group SIM(2) by this additional symmetry. We expect similar results to hold in other scale invariant models that are supplemented only by Lorentz violation with a single lightlike direction.

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- [1] See, e.g., V.A. Kostelecký, ed., *CPT and Lorentz Symmetry III*, World Scientific, Singapore, 2005.
  - [2] S.M. Carroll, G.B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).
  - [3] D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997) [arXiv:hep-ph/9703464]; Phys. Rev. D **58**, 116002 (1998) [arXiv:hep-ph/9809521]; V.A. Kostelecký and R. Lehnert, Phys. Rev. D **63**, 065008 (2001) [arXiv:hep-th/0012060]; V.A. Kostelecký, Phys. Rev. D **69**, 105009 (2004) [arXiv:hep-th/0312310].
  - [4] R. Jackiw and V.A. Kostelecký, Phys. Rev. Lett. **82**, 3572 (1999) [arXiv:hep-ph/9901358]; M. Pérez-Victoria, Phys. Rev. Lett. **83**, 2518 (1999) [arXiv:hep-th/9905061]; B. Altschul, Phys. Rev. D **70**, 101701(R) (2004) [arXiv:hep-th/0407172].
  - [5] F.R. Klinkhamer, Nucl. Phys. B **578**, 277 (2000) [arXiv:hep-th/9912169].
  - [6] C. Adam and F.R. Klinkhamer, Nucl. Phys. B **607**, 247 (2001) [arXiv:hep-ph/0101087].
  - [7] V.A. Kostelecký, R. Lehnert and M.J. Perry, Phys. Rev. D **68**, 123511 (2003) [arXiv:astro-ph/0212003].
  - [8] H. Belich, *et al.* Phys. Rev. D **68**, 065030 (2003) [arXiv:hep-th/0304166].
  - [9] R. Lehnert and R. Potting, Phys. Rev. Lett. **93**, 110402 (2004) [arXiv:hep-ph/0406128]; Phys. Rev. D **70**, 125010 (2004) [arXiv:hep-ph/0408285].
  - [10] B. Feng *et al.*, Phys. Rev. Lett. **96**, 221302 (2006) [arXiv:astro-ph/0601095].
  - [11] R. Jackiw and S.-Y. Pi, Phys. Rev. D **68**, 104012 (2003) [arXiv:gr-qc/0308071].
  - [12] The subgroups of the Poincaré group are classified in, e.g., J. Patera, R.T. Sharp, P. Winternitz, and H. Zassenhaus, J. Math. Phys. **17**, 977 (1976);
  - [13] Our notation follows that of C. Sagiv, N.A. Sochen, and Y.Y. Zeevi, in *Scale-Space 2005*, edited by R. Kimmel, N.A. Sochen, and J. Weickert (Springer-Verlag, Berlin, 2005).
  - [14] A.G. Cohen and S.L. Glashow, Phys. Rev. Lett. **97**, 021601 (2006) [arXiv:hep-ph/0601236].
  - [15] A.G. Cohen and D.Z. Freedman, arXiv:hep-th/0605172.